University of Saskatchewan
Department of Mathematics & Statistics
Mathematics 110.3

Time: 3 hours | Final Examination | 2pm, December 9, 1999

CLOSED BOOK EXAMINATION - NO CALCULATORS ALLOWED

Name: _____ Student #: _____ Math 110 section # _____

PART I

Questions in this part will be marked right or wrong. Please carefully write your answers in the spaces provided.

[7] 1. (a) $\lim_{x\to 2} \pi =$ _____

(b) $\lim_{t\to -1} \dfrac{t+1}{2t^3+7t^2+7t+2} =$ _____

(c) $\lim_{s\to 0} (s^2+1)e^s =$ _____

(d) $\lim_{h\to 0} \dfrac{(x+h)^{10}-x^{10}}{h} =$ _____

(e) $\lim_{\theta\to 0} \theta\csc(2\theta) =$ _____

(f) $\lim_{x\to 1^-} \dfrac{x-1}{|x-1|} =$ _____

(g) $\lim_{x\to -2^+} \dfrac{x}{x^2-4} =$ _____

[3] 2. (a) At what x-value(s) does the graph of $y = \dfrac{x-3}{x^2-4x+3}$ have a vertical asymptote? _____

(b) Find $\lim_{x\to\infty} \dfrac{3x^2-7x+22}{1-x^2}$. _____

(c) Find $\lim_{x\to -\infty} \dfrac{x+2}{\sqrt{4x^2+2x+10}}$. _____

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[20] 3. Carry out the indicated differentiations. It is not necessary to simplify your answers.

(a) If $p(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$,then $p'(x) =$ _____

(b) If $y = \cos(3x)$, then $\frac{dy}{dx} =$ _____

(c) If $w = 2^{(t+2)}$, then $\frac{dw}{dt} =$ _____

(d) If $f(t) = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}$, then $f'(t) =$ _____

(e) If $g(x) = \dfrac{1}{1+x^2}$, then $g'(x) =$ _____

(f) If $u = \dfrac{t^2+1}{t^2+t+1}$, then $\dfrac{du}{dt} =$ _____

(g) If $f(t) = te^t \sin t$, then $f'(t) =$ _____

(h) If $y = \ln\left[\dfrac{x^2-1}{x^2+1}\right]$, then $\dfrac{dy}{dx} =$ _____

(i) If $h(s) = s^{\tan s}$, then $h'(s) =$ _____

(j) If $V = \frac{4}{3}\pi r^3$, then $\frac{dV}{dr} =$ _____

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[10] 4. (a) What is the domain of the function $\log_{10}(4 - x^2)$? _____

(b) Find an antiderivative $F(x)$ of $f(x) = x^2 + \sqrt{x}$ that satisfies $F(1) = 1$. _____

(c) Complete the following definition. A function f is called <u>increasing</u> on an interval I if _____ whenever $x_1 < x_2$ in I.

(d) Complete the following statement: If $f'(x) = 0$ for all x in an interval (a, b), then f is _____ on (a, b).

(e) Complete the following statement: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then _____.

PART II

Please provide carefully written answers to questions 5 through 14 in an answer booklet.

[6] 5. Use the formal definition of the derivative (that is; work from first principles) to find the slope of the tangent line to the graph of $y = 1 - x^2$ at the point $(2, -3)$. (No marks will be given for using the rules of differentiation.)

[6] 6. An oil pipeline under a large lake starts to leak. The oil comes to the surface and forms a growing circular shaped slick with a uniform thickness of 2 cm. At a given time the slick is observed to have a radius of 50 meters and the radius is increasing at a rate of 2 meters per minute. At what rate is the oil leaking from the pipe?

[6] 7. Consider $f(x) = x \ln x$.

(a) What is the domain of $f(x)$? At what point(s) x is $f(x) = 0$?

(b) Identify the intervals where $f(x)$ is increasing or decreasing.

(c) Identify the intervals where $f(x)$ is concave up or down.

(d) Identify any local maxima or minima of $f(x)$.

(e) Sketch a graph of $y = x \ln x$.

(f) Based on your graph, what do you think $\lim_{x \to 0^+} x \ln x$ might be?

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[6] 8. A cylindrical can without a top is required to hold 8π cm^3 of liquid. What is the smallest possible area of material that can be used in making this can? (Assume there is no wastage in constructing the can.)

[6] 9. Find the equation of the tangent line to the graph of $x^{2/3} + y^{2/3} = 5$ at the point $(8, 1)$.

[6] 10. (a) Let $f(\theta) = \cos(2\theta)$. Find $f'(\theta)$, $f''(\theta)$, $f^{(3)}(\theta)$ and $f^{(4)}(\theta)$.

(b) What is $f^{(8)}(0)$?

[6] 11. Use one step of Newton's method to estimate the cube root of 10. That is, let $f(x) = x^3 - 10$ and estimate the root of $f(x)$ by starting with an initial guess of $x_1 = 2$ and applying one step of Newton's method. Leave your answer in fractional form.

[6] 12. Let $f(x) = \dfrac{x}{1+x^2}$ for $x \in [-2, 2]$.

(a) Find all points $x \in [-2, 2]$ that are critical numbers for f.

(b) What are the absolute maximum value and absolute minimum value of $f(x)$ for $x \in [-2, 2]$.

[6] 13. (a) Differentiate the function $G(x) = \dfrac{|x| - 1}{x}$.

(b) Find an antiderivative $F(x)$ of $f(x) = x^{-2}$ that satisfies $F(-1) = 1$ and $F(1) = 1$.

[6] 14. Let c be a constant and $f(x) = x^3 + 3cx^2 + 3x + 2$. Find those values of c for which $f(x)$ has no local maximum. Verify your claim.

The End